The Widom's development on the gauge invariance of the Dicke Hamiltonian

Brief story of an interesting scientific interaction between Prof. Widom and Pirelli Labs

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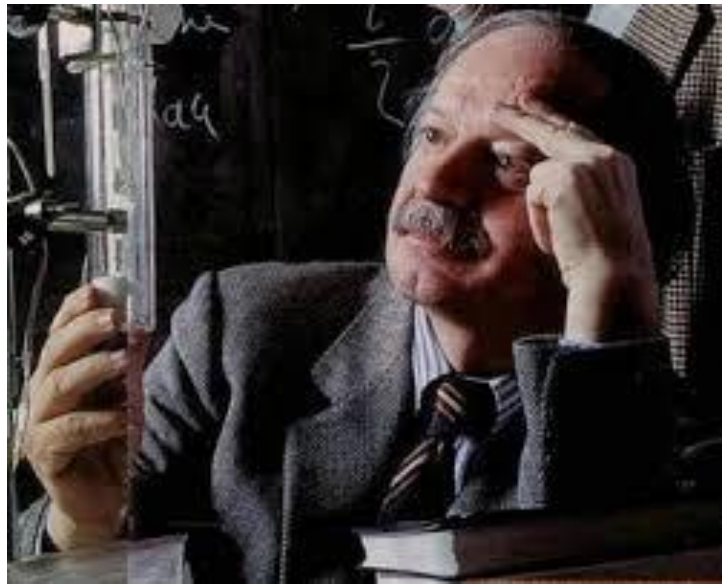
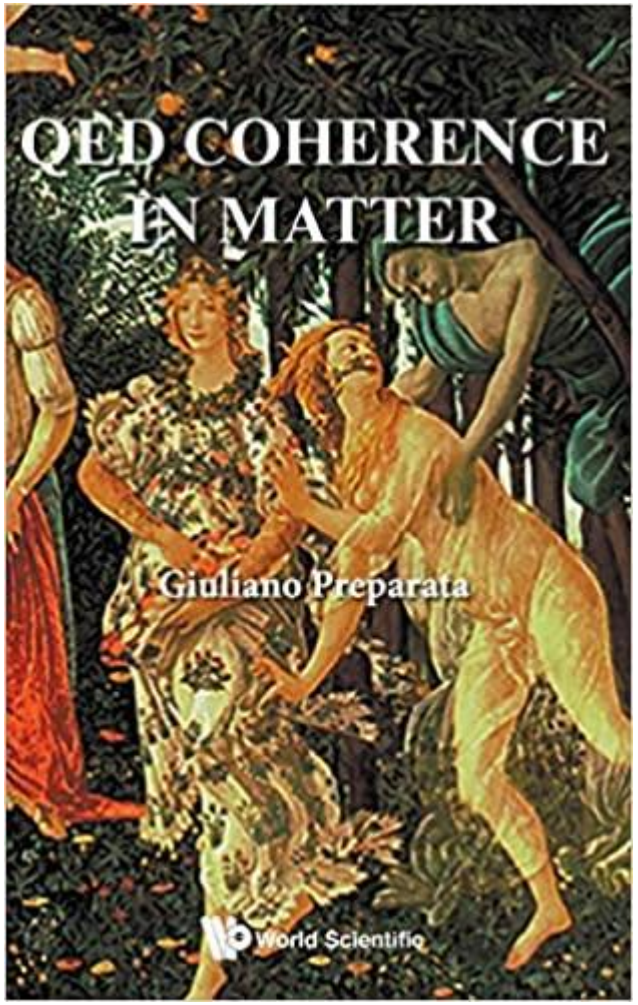


In 1995, the R&D Division of Pirelli Cavi & Systems (later spun off as Pirelli Labs) evaluated the interesting proposal coming from the late Prof. Giuliano Preparata to fund a private laboratory, named LEDA Srl and aimed to demonstrate the possibility of controlling the “cold fusion” phenomena through approaches suggested by Preparata’s theory of coherent electrodynamics in condensed matter

As the manager of the Advanced Research Department at Pirelli Cavi, and as a laser and quantum optics scientist, I was put by the company in charge of the interaction with Prof. Preparata’s team.

The first priority I posed myself was to understand the physics behind Preparata’s model.

NOT A SIMPLE TASK !



- The book of Prof. Preparata, as acknowledged by the author himself, was not a “textbook” but a “research book”- i.e. not conceived to vulgarize the comprehension of his proposal.

- The mathematical treatment was based on the “Path Integral” formalism, typical of Particle Physics but superficially known by other colleagues graduated in Physics and totally unknown to me (a laser scientist and researcher), apart from the introductory lessons of Feynman.

- The need to have a simpler approach, understandable by a larger constituency of physicists and based on an Hamiltonian formulation, was totally overseen by Prof. Preparata.

- (Funny) a lot of quarrels about MKS units refusal

THE QUARREL ON GAUGE INVARIANCE

The theory of Preparata strongly relied on a simplified model of the interaction between e.m. field and an ensemble of atoms, that could be described in terms of creation/annihilation operators of the former and Pauli matrices of the latter in a formalism known as the Dicke Hamiltonian:

$$H = \hbar\omega_c a^\dagger a + \omega_z \sum_{j=0}^{N} \sigma_j^z + \frac{2\lambda}{\sqrt{N}}(a + a^\dagger)\sum_j \sigma_j^x.$$

This approach had been strongly criticized by the Polish physicist Prof. Bialinicki-Birula, who claimed that it was missing the gauge invariance property, which appears fundamental in Physics. A demonstration of this claim was given.

Prof. Preparata succeeded to show that by an iterative calculation the term responsible for the breakdown could be canceled by a series development of the Hamiltonian to higher order in the vector potential A. The proof was nevertheless quite difficult to understand and seemed not to fully solve the issue.

The problem: Pirelli's management was aware of this criticism having prompted the opinion of other experts, and wanted to be sure that the company was not pouring money in a endeavor baseless already at the theoretical level. We had no internal expertise on these exotic topics.

The solution was:

ASK PROF. ALLEN WIDOM!

(Northeastern University- Boston/USA)

Prof. Allen Widom

Prof. Widom immediately become a scientific "guru" of Pirelli Cavi, being:

- **A most talented theoretical physicist**
- **A friendly and somewhat funny guy**
- **A rare example of how difficult physics can be cast in simple equations**
- **A out-of-the-box thinker, by definition**
- **A anti-establishment warrior**

and most importantly
HE LIKED THE PROPOSED PROBLEM A LOT!

How he solved the problem (even outsmarting the original treatment!)

The full treatment can be found in :

Gauge Invariant Formulations of Dicke-Preparata Super-Radiant Models

S. Somu and A. Widom
Physics Department, Northeastern University, Boston MA 02115

Y.N. Srivastava
Physics Department & INFN, University of Perugia, Perugia Italy

August 2000- Physica A: Statistical Mechanics and its Applications 301(1-4)
DOI:10.1016/S0378-4371(01)00384-3

This article, in the opinion of the presenter, is a fundamental landmark in the theory of coherence in condensed matters and could dramatically contribute to its wider comprehension and acceptance.

Another key results – the coherent phase transition

The approach of Prof. Widom is outrageously simple and elegant: by an unitary transformation he converts the Hamiltonian quadratic in the vector potential A into a Hamiltonian linear in the electric field E…… and therefore naturally gauge invariant. The molecular part of the Hamiltonian describing the atoms interacting with the fields:

$$H_{mol}(t) = \sum_{a=1}^{N} K_a \left(\mathbf{p}_a - \frac{q_a}{c}\mathbf{A}(t) \right) + V(\mathbf{r}_1, ..., \mathbf{r}_N)$$

where $K_a(\mathbf{p}_a) = (\mathbf{p}_a^2/2m_a)$ **is a kinetic energy operator, subject to the transformation**

$$U(t) = e^{i(\mu \cdot \mathbf{A}(t)/\hbar c)}$$

becomes: $$\mathcal{H}_{mol}(t) = U^{\dagger}(t) H_{mol}(t) U(t) - i\hbar U^{\dagger}(t) \frac{dU(t)}{dt}$$

$$\mathcal{H}_{mol}(t) = \sum_{a=1}^{N} K_a(\mathbf{p}_a) + V(\mathbf{r}_1, ..., \mathbf{r}_N) - \mu \cdot \mathbf{E}(t)$$

Linear in E !!!

NO NEED OF EXPANSION TO HIGHER ORDERS ! GAUGE INVARIANCE EVIDENT

How he solved the problem – a feat of ingenuity

The transition to coherent phase of Prof. Preparata is obtained by Prof. Widom through a totally different route, based on thermodinamic arguments. Again, the need for a critical density is confirmed.

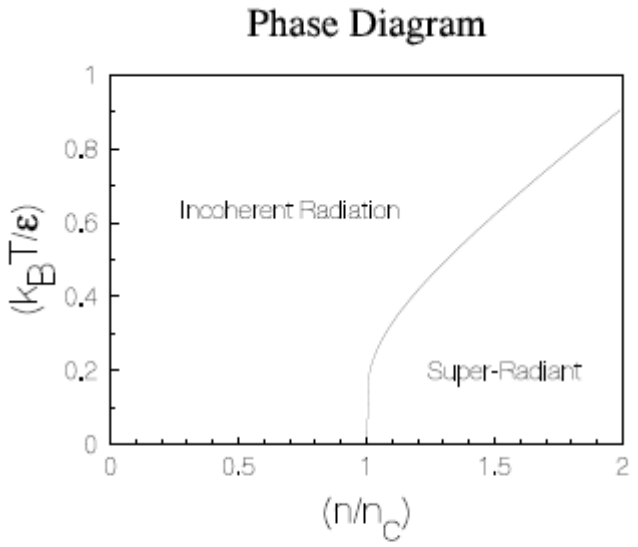


FIG. 2. Shown is the phase diagram in the (n,T)-plane, where n_c is defined in Eq.(66). The two phases are separated by the critical temperature curve $(k_B T_c(n)/\varepsilon)$.

Widom's Hamiltonian vs. Preparata Path Integral: a comparison

Widom's advantages

- Radically simpler treatment
- Easily understandable by other non-particle physicists (laser, superconductivity, solid state etc.) as the formalism is pretty familiar to everybody in the trade

Widom's shortcomings

- No dynamical equations are obtained and solved-> no insight in the runaway process
- Confinement of the e.m. field in matter is not studied
- Renormalization of frequency not indicated explicitly
- Not immediate upgrade to a model with spatial dependence
- No hint on "coherent domain" dimensions

“Where are all the flowers gone?”

As most of the Italian high-tech and industrial R&D, also Pirelli Labs were washed away by the financial bubble around 2007-2008.

“If an industry seeks resources to enter finance/stock trading, the first costs to cut are the curvaceous receptionist and R&D “

(wisdom of the time)

Some of the Preparata’s (and Widom’s) ideas are still object of investigation thanks to the forward-looking attitude of Dr. Milly Moratti (SARAS Group) who collaborates with the brilliant Dr. Luca Gamberale, one of the most appreciated Ph.D. student of Prof. Preparata.
The presenter’s company (NOGAH PHOTONICS- Israel) keeps open a scouting activity on some topics as well

We wish Happy Birthday to Prof. Widom, and we consider ourselves lucky and honored of having had the opportunity to “trigger” his attention towards this intriguing problem.

Thanks for your attention
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